Taking Location Seriously: Is Location a Function or a Relation?

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Abstract. The focus of the paper is 'location as a function' vs 'location as a relation'. To this end, I develop as a logical theory a high-level description of a data model with two embedding spaces, one Euclidean and one topological, and then show how by using a 'mirroring' relation the structure that a spatial object takes in one embedding space constrains the structure it can take in the other. In conclusion, though, it is found that what can be done with two embedding spaces can actually be done with one, providing that the single embedding space is supplemented with different kinds of locational relations.

1 LOCATION AS A FUNCTION

1.1 The Classical Model

Spatial objects distinct from the regions in which they are located are rarely considered in GIS. Figure 1 shows an example of what is a typical view of a GIS data model. It shows a vector-based map in which each geometric construction is associated with a spatial object. Were this view to form the basis of a data viewer application, a user might have the option of selecting one of the geometric constructions using a mouse. A pop-up window would then display details of all the spatial object's 'non-spatial' attributes (e.g., its owner, category, and so forth). This is a familiar picture and lends itself to the misconceived view of the 'GIS as just computer map.'

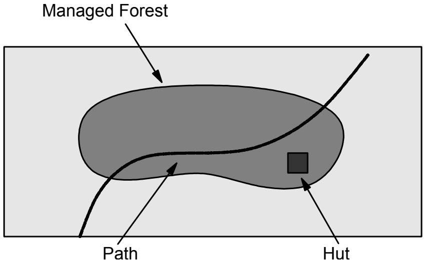

Figure 1: Spatial database as computer map

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A slightly more sophisticated view of the same data model—a data model that shall henceforth be called the 'classical model'—is illustrated in Figure 2. In this figure, objects are separated from their locations and populate an 'information space', while their locations are regions ('precise regions') in a 'precise space' (or coordinate reference system). The precise space of a typical GIS is the coordinatised, Euclidean plane.

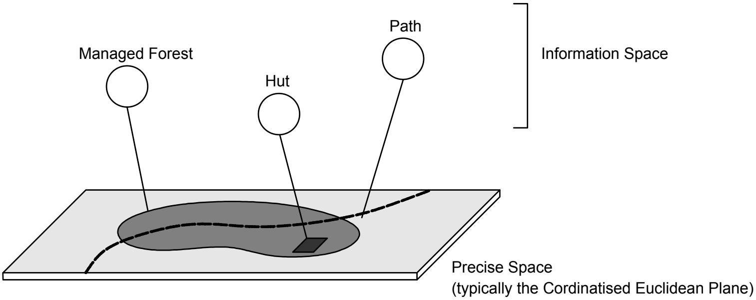

Figure 2: The classical model

Now there is a problem with the classical model. For each spatial object *o* in information space, there must be some unique, minimal precise region *R* in which *o* is located. (Or as we might put it, spatial objects must be 'exactly located' or must have 'exact location'.) Yet many spatial objects that we might want to have in information space do not have exact location. That is, because of either deficiencies in our knowledge or intrinsic vagueness in the object itself, the locations of these objects cannot be specified by an exact set of coordinates. For example, a rare species of butterfly is known to inhabit the managed forest. We know the population is definitely within the forest; but it is in inappropriate to say that the location of the population is the same as or equal to the forest's location. Ideally we just want to represent the fact 'the butterfly population is located within the managed forest'. Yet without assigning the population a precise region, the fact cannot be represented in the classical model.

This problem ('the problem of indeterminate location') is not a new one for GIScience (see, for example, the book (Burrough 1996). Yet its proper formalization can only be given when we tease the objects away from the regions in which they are located. This is the first step in taking location seriously. The next step is to analyse the kind of association that location is. Is it, for example, a function or is it a relation? Each of these possibilities will now be explored as a solution to the problem.

1.2 Multiple Embedding Spaces

There are many kinds of embedding space, of which Precise Space is just one. The following is a brief sketch for supplementing precise space with T-Space, another kind of embedding space.

An embedding space is characterised by the group of transformations under which a set of propositions remain true (Worboys 2004), where the set of propositions in question correspond to the possible facts representable in the embedding space. In a topological embedding space (or T-Space), for example, the set of transformations are the topological ('rubber-sheet') transformations. Under these transformations some propositions remain true, e.g., 'the hut is located within the forest', 'the path crosses the forest'; but other propositions do not, e.g., 'the stretch of the path which intersects the forest is 5 miles long', since distance is not invariant under topological transformation. The fact 'the butterfly population is located within the forest' can be represented in T-Space as follows. Let *b* and *f* denote the butterfly population and the managed forest. Let loc_T denote the function which maps objects to their embeddings (locations) in T-Space. Then, assuming an underlying mereotopological theory to govern T-Space, we can represent the fact explicitly as $\mathsf{P}(loc_T(b), loc_T(f))$, where P denotes parthood.

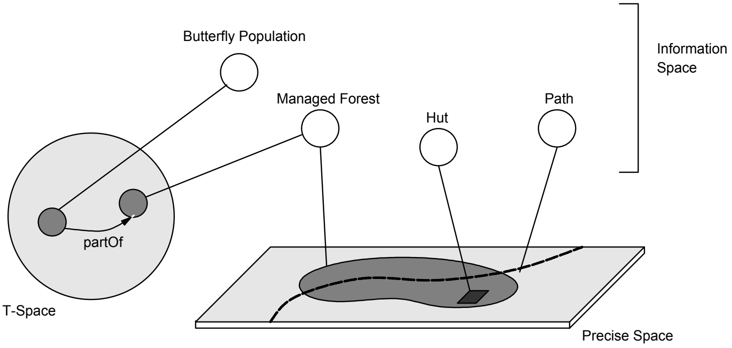

Figure 3: Multiple embedding spaces

Having multiple embedding spaces is all very well, but what are the structural connections between them? Let us suppose the database system is posed the query, 'Is there a butterfly population within 5 miles from the hut?' Clearly we can see that there is such a population, given that the path through the forest is 5 miles long. However, without being told how relationships between objects in one space constrain relationships between objects in the other, the database cannot. What connections can we add to make reasoning across different spaces possible?

The necessary connection we seek I call ‘conditional mirroring’. Let φ be any spatial relation of a mereotopological theory shared by both precise space and T-Space. Let L denote exact location, so that $\mathsf{L}(o,R)$ means ‘o is exactly located on R’. Then conditional mirroring is given by the rule

CM. $\mathsf{L}(x,X) \;\&\; \mathsf{L}(y, Y) \rightarrow (\varphi(X, Y) \;\&\; (loc_T(x), loc_T(y)))$

This says (roughly) that, if two objects are exactly located, than their spatial properties are mirrored in the two embedding spaces. With reference to our query, let r denote the spatial object of our query region—that is, the circle of radius 5 miles centred on the hut. Let R denote the representation of the query region in precise space, so that r is exactly located on the precise region R. Let F and B be the regions on which f and b are exactly located. Now, by conditional mirroring and the facts $\mathsf{P}(F,R)$, $\mathsf{L}(r,R)$ and $\mathsf{L}(f, F)$, we get $\mathsf{P}(loc_T(f), loc_T(r))$. But then from this, $\mathsf{P}(loc_T(b), \mathrm{loc}_T(f))$ and transitivity of parthood, we get $\mathsf{P}(loc_T(b), loc_T(r))$. Hence, the database system concludes that the butterfly population is located within the query region.

2 LOCATION AS A RELATION

An alternative to the above is offered by Anchoring (Hood 2006). In Anchoring location is not a function, but a relation—it is a many-to-many relation from objects to regions. There are many different kinds of locational relation and an object can be related to a region in many different ways. Furthermore, the behaviour of the different locational relations is governed by a small set of axioms which can replicate the reasoning we saw above. For example, the butterfly population is ‘anchored in’ the precise region of the managed forest F. That is to say, whatever may be the nature of b’s location, it is a fact that b is located within F.

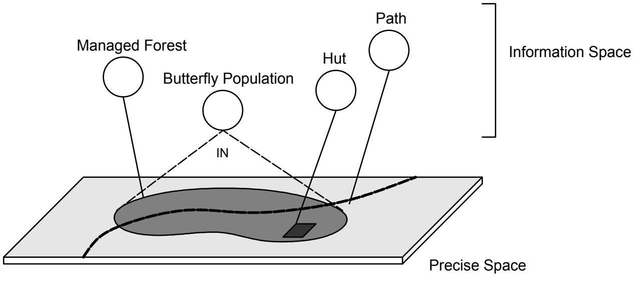

Figure 4: Anchoring

3 CONCLUSION

In the multiple embedding spaces model, despite there being two embedding spaces and thus two location functions, location is still a kind of function. In the Anchoring model, by contrast, location is a relation. Which one of these options do we choose? Space has not permitted a complete analysis and comparison of the two approaches, which is something for future work. Yet one thing is for sure. If we are serious about building good spatial data models, location must not be taken for granted.

References

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